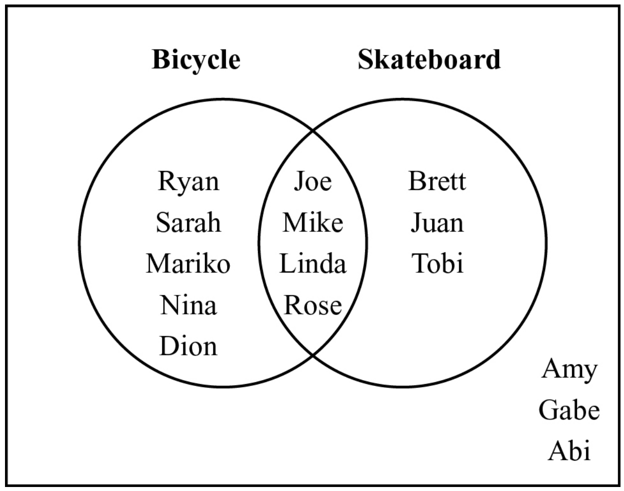
REVIEW EXAMPLES

1) This Venn diagram shows the names of students in Mr. Leary's class that own bicycles and skateboards.



Let set A be the names of students who own bicycles, and let set B be the names of students who own skateboards.

a. Find $A \cap B$. What does the set represent?

b. Find $A \cup B$. What does the set represent?

c. Find $(A \cup B)'$. What does the set represent?

Solution:

a. The intersection is the set of elements that are common to both set A and set B, so $A \cap B$ is {Joe, Mike, Linda, Rose}. This set represents the students who own both a bicycle and a skateboard.

b. The union is the set of elements that are in set A or set B, or in both set A and set B. You only need to list the names in the intersection one time, so $A \cup B$ is {Ryan, Sarah, Mariko, Nina, Dion, Joe, Mike, Linda, Rose, Brett, Juan, Tobi}. This set represents the students who own a bicycle, a skateboard, or both.

c. The complement of $A \cup B$ is the set of names that are not in $A \cup B$. So, $(A \cup B)'$ is {Amy, Gabe, Abi}. This set represents the students who own neither a bicycle nor a skateboard.

2) In a certain town, the probability that a person plays sports is 65%. The probability that a person is between the ages of 12 and 18 is 40%. The probability that a person plays sports and is between the ages of 12 and 18 is 25%. Are the events independent? How do you know?

Solution:

Let $P(S)$ be the probability that a person plays sports.
Let $P(A)$ be the probability that a person is between the ages of 12 and 18.
If the two events are independent, then $P(S \cup A) = P(S) \bullet P(A)$.
Because $P(S \cup A)$ is given as 25%, find $P(S) \bullet P(A)$ and then compare.

$$\begin{aligned} P(S) \bullet P(A) &= 0.65 \bullet 0.4 \\ &= 0.26 \end{aligned}$$

Because $0.26 \neq 0.25$, the events are not independent.

3) A random survey was conducted to gather information about age and employment status. This table shows the data that were collected.

Employment Survey Results

	Age (in Years)	
Employment Status	**Less than 18**	**18 or greater**
Has Job	20	587
Does Not Have Job	245	92

a. What is the probability that a randomly selected person surveyed has a job, given that the person is less than 18 years old?

b. What is the probability that a randomly selected person surveyed has a job, given that the person is greater than or equal to 18 years old?

c. Are having a job (A) and being 18 or greater (B) independent events? Explain.

Solution:

a. Find the total number of people surveyed less than 18 years old: 20 + 245 = 265. Divide the number of people who have a job and are less than 18 years old, 20, by the number of people less than 18 years old, 265: $\frac{20}{265} \approx 0.08$. The probability that a person surveyed has a job, given that the person is less than 18 years old is about 0.08.

b. Find the total number of people surveyed greater than or equal to 18 years old: 587 + 92 = 679. Divide the number of people who have a job and are greater than or equal to 18 years old, 587, by the number of people greater than or equal to 18 years old, 679: $\frac{587}{679} \approx 0.86$. The probability that a person surveyed has a job, given that the person is greater than or equal to 18 years old, is about 0.86.

c. The events are independent if $P\left(A \mid B\right) = P(A)$ and $P\left(B \mid A\right) = P(B)$.

From part (b), $P\left(A \mid B\right) \approx 0.86$.

$P(A) = \frac{607}{944} \approx 0.64$

$P\left(A \mid B\right) \neq P(A)$ so the events are not independent.

REVIEW EXAMPLES

1) In Mr. Mabry's class, there are 12 boys and 16 girls. On Monday, 4 boys and 5 girls were wearing white shirts.

   a. If a student is chosen at random from Mr. Mabry's class, what is the probability of choosing a boy or a student wearing a white shirt?

   b. If a student is chosen at random from Mr. Mabry's class, what is the probability of choosing a girl or a student not wearing a white shirt?

Solution:

a. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where A is the set of boys and B is the set of students wearing a white shirt.

   A and B is the set of boys wearing a white shirt. There are 12 + 16 = 28 students in Mr. Mabry's class.

   So, $P(A) = \frac{12}{28}$, $P(B) = \frac{4+5}{28} = \frac{9}{28}$, and $P(A \text{ and } B) = \frac{4}{28}$.

   $P(\text{a boy or a student wearing a white shirt}) = \frac{12}{28} + \frac{9}{28} - \frac{4}{28} = \frac{17}{28}$.

b. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, where A is the set of girls and B is the set of students not wearing a white shirt.

   A and B is the set of girls not wearing a white shirt. There are 12 + 16 = 28 students in Mr. Mabry's class.

   So, $P(A) = \frac{16}{28}$, $P(B) = \frac{8+11}{28} = \frac{19}{28}$, and $P(A \text{ and } B) = \frac{11}{28}$.

   $P(\text{a girl or a student not wearing a white shirt}) = \frac{16}{28} + \frac{19}{28} - \frac{11}{28} = \frac{24}{28} = \frac{6}{7}$.

2) Terry has a number cube with sides labeled 1 through 6. He rolls the number cube twice.

a. What is the probability that the sum of the two rolls is a prime number, given that at least one of the rolls is a 3?

b. What is the probability that the sum of the two rolls is a prime number or at least one of the rolls is a 3?

Solution:

a. This is an example of a mutually exclusive event. Make a list of the combinations where at least one of the rolls is a 3. There are 11 such pairs.

1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
3, 1	3, 2	3, 4	3, 5	3, 6	

Then identify the pairs that have a prime sum.

2, 3	3, 2	3, 4	4, 3

Of the 11 pairs of outcomes, there are 4 pairs whose sum is prime. Therefore, the probability that the sum is prime of those that show a 3 on at least one roll is $\frac{4}{11}$.

b. This is an example of events that are NOT mutually exclusive. There are 36 possible outcomes when rolling a number cube twice.

List the combinations where at least one of the rolls is a 3.

1, 3	2, 3	3, 3	4, 3	5, 3	6, 3
3, 1	3, 2	3, 4	3, 5	3, 6	

$P(\text{at least one roll is a 3}) = \frac{11}{36}$

List the combinations that have a prime sum.

1, 1	1, 2	1, 4	1, 6
2, 1	2, 3	2, 5	
3, 2	3, 4		
4, 1	4, 3		
5, 2	5, 6		
6, 1	6, 5		

$P(\text{prime sum}) = \frac{15}{36}$