Unit 7: Applications of Probability

This unit investigates the concept of probability. Students look at sample spaces and identify unions, intersections, and complements. They identify ways to tell if events are independent. The concept of conditional probability is related to independence and students use the concepts to solve real-world problems, including those that are presented in two-way frequency tables. Students find probabilities of compound events using the rules of probability.

KEY STANDARDS

Understand independence and conditional probability and use them to interpret data

MCC9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★

MCC9-12.S.CP.2 Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★

MCC9-12.S.CP.3 Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.★

MCC9-12.S.CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★

MCC9-12.S.CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★

Use the rules of probability to compute probabilities of compound events in a uniform probability model

MCC9-12.S.CP.6 Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.★

MCC9-12.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★

UNDERSTAND INDEPENDENCE AND CONDITIONAL PROBABILITY AND USE THEM TO INTERPRET DATA

KEY IDEAS

1. In probability, a ***sample space*** is the set of all possible outcomes. Any subset from the sample space is an ***event***.

2. If the outcome of one event does not rely on the other event, the events are ***independent***. If the outcome of one event relies on the other event, the events are ***dependent***.

3. The ***intersection*** of two or more events is all of the outcomes shared by both events. The intersection is denoted with the word "and," or with the $\cap$ symbol. For example, the intersection of A and B is shown as $A \cap B$.

4. The ***union*** of two or more events is all of the outcomes for either event. The union is denoted with the word "or," or with the $\cup$ symbol. For example, the union of A and B is shown as $A \cup B$. The probability of the union of two events that have no outcomes in common is the sum of each individual probability.

5. The ***complement*** of an event is the set of outcomes in the same sample space that are not included in the outcomes of the event. The complement is denoted with the word "not," or with the $'$ symbol. For example, the complement of A is shown as A'. The set of outcomes and its complement make up the entire sample space.

6. ***Conditional probabilities*** are found when one event has already occurred and a second event is being analyzed. Conditional probability is denoted $P(A \mid B)$ and is read as "The probability of A given B."

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}$$

7. Two events—A and B—are independent if the probability of the union is the same as the product of each individual probability. That is, $P(A \cup B) = P(A) \bullet P(B)$.

8. If two events are independent, then $P(A \mid B) = P(A)$ and $P(B \mid A) = P(B)$.

9. ***Two-way frequency tables*** summarize data in two categories. These tables can be used to show if the two events are independent and to approximate conditional probabilities.

Example:

A random survey was taken to gather information about grade level and car ownership status of students at a school. This table shows the results of the survey.

Car Ownership by Grade

	Owns a Car	Does Not Own a Car	Total
Junior	6	10	16
Senior	12	8	20
Total	18	18	36

Estimate the probability that a randomly selected student will be a junior, given that the student owns a car.

Solution:

Let $P(J)$ be the probability that the student is a junior. Let $P(C)$ be the probability that the student owns a car.

$$P(J \mid C) = \frac{P(J \text{ and } C)}{P(C)} = \frac{\frac{6}{36}}{\frac{18}{36}} = \frac{6}{18} = \frac{1}{3}$$

The probability that a randomly selected student will be a junior given that the student owns a car is $\frac{1}{3}$.

USE THE RULES OF PROBABILITY TO COMPUTE PROBABILITIES OF COMPOUND EVENTS IN A UNIFORM PROBABILITY MODEL

KEY IDEAS

1. Two events are ***mutually exclusive*** if the events cannot occur at the same time.

2. When two events A and B are mutually exclusive, the probability that event A or event B will occur is the sum of the probabilities of each event: $P(A \text{ or } B) = P(A) + P(B)$.

3. When two events A and B, are not mutually exclusive, the probability that event A or B will occur is the sum of the probability of each event minus the overlap of the two events. That is, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.

4. You can find the conditional probability, $P(A \mid B)$, by finding the fraction of B's outcomes that also belong to A.

   Example:

   Event A is choosing a heart card from a standard deck of cards.
   Event B is choosing a face card from a standard deck of cards.
   $P(A \mid B)$ is the probability that a card is a heart, given that the card is a face card. You can look at B's outcomes and determine what fraction belongs to A; there are 12 face cards, 3 of which are also hearts:
   $P(A \mid B) = \frac{3}{12} = \frac{1}{4}$.

	Heart	Not a heart	Total
Face card	3	9	12
Not a face card	10	30	40
Total	13	39	52